

國立虎尾科技大學九十九學年度研究所（碩士班）入學考試試題

所別：機械設計工程系碩士班

科目：考試科目（工程數學）

注意事項：

(1) 共六大題，第一至第四題每題十五分，第五及第六題每題二十分；共一百分。

(2) 請於答案卷上註明題號。

1. Please provide the definitions of the following terms: (15%)

(a) Ordinary Differential Equation (5%)

(b) Laplace Transform (5%)

(c) Vector (5%)

2. Find the eigenvalues and eigenvectors of matrix A for the linear system $AX = B$, where

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} 1 \\ 3 \end{bmatrix}. \quad (15\%)$$

3. Transform the quadratic form $x_1^2 + 6x_1x_2 + x_2^2 = 0$ to the principal axes. Express the new coordinate vector $\mathbf{y}^T = [y_1 \ y_2]$ in terms of the original $\mathbf{x}^T = [x_1 \ x_2]$. (15%)

4. Find the volume of the hexahedron with vertices: $(2,3,5), (4,3,2), (1,2,3), (3,2,1)$. (15%)

5. Solve the following initial value problem. (20%)

$$4y''(x) + 4y'(x) + 17y(x) = 0, \quad y(0) = -1, \quad y'(0) = 2.$$

6. Solve the given problem by Laplace transform. (20%)

$$y''(t) + 4y'(t) + 3y(t) = 6, \quad y(0) = 0, \quad y'(0) = 0.$$

(a) Find the Laplace transform of the differential equation. (10%)

(b) Calculate the response $y(t)$ by inverse Laplace transform. (10%)